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Nuclear energy gap calculations using realistic and delta-function forces

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Abstract. Energy gap calculations are made with the realistic Yale–Shakin interaction and the standard δ-function interaction. States below the valence shell are taken into account. It is found that the results are very close, showing that they are practically insensitive to the details of the interaction. The constant-Δ approximation is found to be very well justified.

1. Introduction

One of the major problems of nuclear physics has been the precise knowledge of the nucleon–nucleon interaction. Considerable interest has been aroused by Brown and his collaborators (e.g. Kuo and Brown 1966) in the use of the realistic interactions in nuclear structure calculations. Recently there have also been attempts to bypass the problem of the potential and to derive the two-body matrix elements directly from the phase-shift data (Elliott *et al.* 1968). It has been found that a number of low-energy nuclear properties are not very sensitive to the detailed form of the interaction.

The purpose of this work is to study two types of interactions for the problem of pairing. One is the 'realistic' Yale interaction (Lassila *et al.* 1962), whose matrix elements have recently become available through the work of Shakin and his collaborators (1966 a,b), and the other is the standard δ-function interaction $J\delta(\boldsymbol{r})$, where J is the strength of the force. The Yale potential is a complicated finite-range interaction with a hard core, and the Yale–Shakin matrix elements have been derived using the separation technique (Moszkowski and Scott 1960), according to which the very short-range part of the potential does not contribute to the energy. Pairing, however, is known to be a short-range force and the δ interaction is found to satisfy the requirements of a pairing potential (Lane 1964, chap. 1). It will, therefore, be interesting to compare the results for the two interactions. Brown and Kuo (1967) have also given arguments to show that the 'bare' interaction acts as a short-range force.

In a previous paper (Jain and Waghmare 1969, to be referred to as I) we presented our detailed calculations of energy-gap functions and pairing energies for the two potentials. It was found that the results are very close. This paper presents the results of further calculations. The strength J of the δ force is derived from the odd–even mass difference data and is compared with the corresponding values obtained from the matrix elements derived from the Talmi approach (Talmi 1962).

2. The gap equations

The energy-gap equation is given by (Lane 1964, chap. 3)

$$\Delta_j = -\tfrac{1}{2}\sum_{j'} \langle (j^2)\mathrm{O}^+ | v | (j'^2)\mathrm{O}^+ \rangle \frac{\{j'\}}{\{j\}} \frac{\Delta_{j'}}{\{(\epsilon_{j'}-\lambda)^2+\Delta_{j'}^2\}^{1/2}} \qquad (1)$$

where $\{j\} = (2j+1)^{1/2}$, v represents the two-body interaction, λ is the chemical potential and the ϵ_j are the self-consistent energies of the single-particle states j. For the δ interaction and the oscillator wave functions, we have

$$\langle (j^2)\mathrm{O}^+ | -J\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) | (j'^2)\mathrm{O}^+ \rangle = -\frac{J\nu^{3/2}}{8\pi}\{j\}\{j'\}I_{nl,n'l'} \tag{2}$$

where $I_{nl,n'l'}$ is the radial integral in units of $\nu^{3/2}$, ν being the oscillator parameter $m\omega/\hbar$. This leads to

$$\Delta_j = \frac{J\nu^{3/2}}{8\pi}\sum_{j'}\left(\frac{2j'+1}{2}\right)\frac{I_{nl,n'l'}\Delta_{j'}}{\{(\epsilon_{j'}-\lambda)^2+\Delta_{j'}{}^2\}^{1/2}}. \tag{3}$$

One can obtain the 'idealized' constant-Δ equation on replacing $I_{nl,n'l'}$ by some average I_{av}

$$1 = \frac{JI_{\mathrm{av}}\nu^{3/2}}{8\pi}\sum_{j}\left(\frac{2j+1}{2}\right)\frac{1}{\{(\epsilon_j-\lambda)^2+\Delta^2\}^{1/2}}. \tag{4}$$

Equation (1), (2) or (3) is to be solved in conjunction with the 'number-conservation' equation

$$N = 2\sum_{j}\left(\frac{2j+1}{2}\right)V_j{}^2 \tag{5}$$

where $V_j{}^2$ is the occupation probability of the pair state j

$$V_j{}^2 = \tfrac{1}{2}\left[1 - \frac{\epsilon_j-\lambda}{\{(\epsilon_j-\lambda)^2+\Delta_j{}^2\}^{1/2}}\right]. \tag{6}$$

3. Calculations and results

For the Yale–Shakin interaction, the two-body matrix elements are obtained by using the tables of relative matrix elements given by Shakin *et al.* (1966 b) and the expression (18) of their paper (1966 a). These are then substituted in equation (1) and the gap equations are solved for Δ and λ. The major problem in solving the gap equations is to know the values of ϵ_j. These quantities can only be evaluated numerically in a self-consistent way. The procedure is highly painstaking and, therefore, we have not undertaken this task in the present calculation. We use the observed single-proton and single-neutron excitations tabulated by Sherwood and Waghmare (1965). It is found that the results are not very sensitive to the choice of single-particle energies. Moreover, the main purpose of this work is to compare the results of the two types of potentials, and since the same set of single-particle energies is being used for both, the error introduced will be insignificant.

The strength J of the δ force is derived by solving the gap equations for various values of J. The values of J which reproduce the experimental pairing energies are selected. The gap equations were solved for both neutron and proton pairing energies. The J values obtained from neutron energy-gap calculations for some of the nuclei are presented in table 1. The values for proton calculations are quite similar.

The first successful calculations based on pairing theory were made by Kisslinger and Sorensen (1960). In I we compared the J values derived from the Kisslinger–Sorensen parameter G with the J values obtained by us. The agreement was found to be good. Various authors have derived information about the effective two-body matrix elements from experimental spectra, following the Talmi approach. If one

Table 1. Values of the strength parameter J required to obtain the observed neutron-pairing energies using the δ interaction (in units of MeV fm^3)

Isotopes	Mass number	J for constant-Δ solution	J for state-dependent Δ solution
	16	197·6	180·0
Oxygen	18	160·2	180·0
	20	154·0	174·0
	34	221·7	210·0
Sulphur	36	237·8	230·0
	38	209·4	227·0
	42	209·1	227·0
Calcium	44	212·4	232·0
	46	197·1	218·0
	60	214·3	220·0
	62	229·4	228·0
Nickel	64	229·4	228·0
	66	208·0	210·0
	92	242·0	249·0
Zirconium	94	237·8	240·0
	96	231·1	220·0

Table 2. J values derived from the Talmi matrix elements

Nuclei	Matrix element ($J = 0$, $T = 1$)	Parametrized value (absolute) (MeV)	J value (derived) (MeV fm^3)	Mean J (MeV fm^3)
Oxygen isotopes (Federman–Talmi 1965)	$\langle(0d_{5/2})^2\|v\|(0d_{5/2})^2\rangle$	3·24	266·3	216·7
	$\langle(1S_{1/2})^2\|v\|(1S_{1/2})^2\rangle$	1·97	199·7	
	$\langle(0d_{5/2})^2\|v\|(1S_{1/2})^2\rangle$	0·77	184·2	
$28 \leqslant A \leqslant 40$ (Glaudemans *et al.* 1964)	$\langle(0d_{3/2})^2\|v\|(0d_{3/2})^2\rangle$	2·28	397·7	295·7
	$\langle(1S_{1/2})^2\|v\|(1S_{1/2})^2\rangle$	1·351	193·6	
^{33}S–^{41}Ca (Erne 1966)	$\langle(0d_{3/2})^2\|v\|(0d_{3/2})^2\rangle$	1·71	311·1	311·1
Calcium isotopes (Federman–Talmi 1966)	$\langle(0f_{7/2})^2\|v\|(0f_{7/2})^2\rangle$	2·64	355·4	388·7
	$\langle(1p_{3/2})^2\|v\|(1p_{3/2})^2\rangle$	1·40	255·2	
	$\langle(0f_{7/2})^2\|v\|(1p_{3/2})^2\rangle$	1·64	555·4	
Nickel isotopes (Auerbach 1966)	$\langle(0f_{5/2})^2\|v\|(0f_{5/2})^2\rangle$	1·74	374·9	360·6
	$\langle(0f_{5/2})^2\|v\|(1p_{3/2})^2\rangle$	1·12	515·9	
	$\langle(0f_{5/2})^2\|v\|(1p_{1/2})^2\rangle$	0·56	371·8	
	$\langle(1p_{3/2})^2\|v\|(1p_{3/2})^2\rangle$	0·92	201·3	
	$\langle(1p_{3/2})^2\|v\|(1p_{1/2})^2\rangle$	0·97	300·3	
	$\langle(1p_{1/2})^2\|v\|(1p_{1/2})^2\rangle$	0·89	389·5	
^{90}Zr (Talmi–Unna 1960)	$\langle(0g_{9/2})^2\|v\|(1p_{1/2})^2\rangle$	0·708	471·7	421·35
	$\langle(0g_{9/2})^2\rangle - \langle(1p_{1/2})^2\rangle$	1·164	371·0	

Table 3. Occupation probabilities of neutron states in Ni and Zr isotopes

Nucleus	State	Occupation probability (V_j^2) for			Experimental values
		Yale–Shakin interaction	δ interaction (state-dependent solution)	δ interaction (constant solution)	
	$0f_{7/2}$	0·939	0·935	0·940	
	$1p_{3/2}$	0·628	0·607	0·603	0·775
^{60}Ni	$0f_{5/2}$	0·284	0·287	0·274	0·15
	$1p_{1/2}$	0·062	0·05	0·049	
	$0g_{9/2}$	0·020	0·026	0·030	
	$0f_{7/2}$	0·942	0·943	0·943	
	$1p_{3/2}$	0·835	0·746	0·736	
	$0f_{5/2}$	0·44	0·472	0·468	
^{62}Ni	$1p_{1/2}$	0·081	0·091	0·096	
	$0g_{9/2}$	0·031	0·044	0·056	
	$1d_{5/2}$	0·003	0·011	0·019	
	$0f_{7/2}$	0·963	0·962	0·962	
	$1p_{3/2}$	0·920	0·858	0·847	
	$0f_{5/2}$	0·650	0·658	0·656	
^{64}Ni	$1p_{1/2}$	0·11	0·128	0·136	
	$0g_{9/2}$	0·046	0·059	0·072	
	$1d_{5/2}$	0·004	0·013	0·022	
	$0f_{7/2}$	0·984	0·987	0·987	
	$1p_{3/2}$	0·960	0·958	0·954	
	$0f_{5/2}$	0·86	0·888	0·886	
^{66}Ni	$1p_{1/2}$	0·215	0·171	0·174	
	$0g_{9/2}$	0·06	0·055	0·066	
	$1d_{5/2}$	0·004	0·008	0·014	
	$0f_{5/2}$	0·998	0·996	0·994	
	$1p_{1/2}$	0·988	0·982	0·985	
	$0g_{9/2}$	0·996	0·984	0·972	
^{92}Zr	$1d_{5/2}$	0·324	0·344	0·363	0·30
	$2s_{1/2}$	0·024	0·042	0·052	0·05
	$1d_{3/2}$	0·012	0·01	0·010	0·025
	$0g_{7/2}$	0·001	0·002	0·004	0·03
	$0f_{5/2}$	0·999	0·998	0·996	
	$1p_{1/2}$	0·989	0·989	0·991	
	$0g_{9/2}$	0·997	0·993	0·984	
^{94}Zr	$1d_{5/2}$	0·655	0·659	0·664	0·62 ± 0·5
	$2s_{1/2}$	0·045	0·071	0·078	0·05
	$1d_{3/2}$	0·018	0·010	0·010	0·05
	$0g_{7/2}$	0·001	0·002	0·004	0·04
	$0f_{5/2}$	1·000	1·000	1·000	
	$1p_{1/2}$	1·000	0·999	1·000	
	$0g_{9/2}$	1·000	0·999	1·000	
^{96}Zr	$1d_{5/2}$	0·994	0·981	0·993	0·96 ± 0·10
	$2s_{1/2}$	0·016	0·064	0·013	0·05
	$1d_{3/2}$	0·001	0·002	0·001	0·05
	$0g_{7/2}$	0·000	0·000	0·000	0·06

assumes a δ force, one can easily obtain the relation

$$J = \frac{8\pi}{\{j\}\{j'\}I_{nl,n'l'}\nu^{3/2}}|V|$$

where $|V|$ is the Talmi parameter $|\langle(j^2)O^+|v|(j'^2)O^+\rangle|$. We have derived the J values from some of the Talmi parameters and the results are given in table 2. It can be seen that these J values are consistently higher than our results. This is what one would expect, firstly, because the number of single-particle states used in the Talmi approach is much smaller than that used by Kisslinger–Sorensen and ourselves. This increases the 'effective' force. Secondly, these matrix elements have been obtained by requiring a detailed fit to nuclear spectra, and it has been observed that the matrix elements required to give a detailed fit to nuclear data are higher than the Kisslinger–Sorensen matrix elements (e.g. Brown 1967).

Using the J values given in table 1, we have calculated the occupation probabilities V_j^2 for the single-particle states j in Ni and Zr isotopes. These are given in table 3 with the corresponding values calculated with the Yale–Shakin interaction. Also given for comparison are the experimental results obtained from (d, p) and (d, t) reactions (Cohen and Price 1961, Macfarlane *et al.* 1962). It is seen that the results for the Yale–Shakin force and for the δ force are essentially the same and agree reasonably well with the experimental results. These results, with the results of I, show clearly that the δ force closely simulates the Yale–Shakin realistic interaction, and that the realistic interaction works very well as a 'pairing' force in spite of the fact that the very short-range part does not contribute to the energy.

In our calculations we have included the states below the 'valence' shell. Thus the effects of core polarization arising from 2p–2h excitations of the core are taken care of.

Finally, we note that there is no difference between the results of state-dependent and constant-Δ solutions for the δ force, showing that the 'idealized' pairing force model, with which the earlier authors had to work because of a lack of knowledge of the nature of the force, is very well justified.

4. Conclusions

The major conclusion of this work is that the δ-function interaction closely simulates the 'pairing' part of a 'realistic' finite-range interaction of complicated nature. This supports the fact that many of the low-energy properties of nuclei are not sensitive to the details of the interaction. We also note that the constant-Δ approximation of the gap equations is very well justified.

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